1 a) slides p6 $\frac{d\phi}{dx} = \frac{\phi_{i+1} - \phi_{i-1}}{2h} + \frac{h^2}{6}\frac{d^3\phi(\xi_i)}{dx^3} + \cdots$

$\Rightarrow$ for $u\frac{d\phi}{dx}$: $\tau = u\frac{h^2}{6}\frac{d^3\phi(\xi_i)}{dx^3}$

B.C. at $x=1$ same derivative $\longrightarrow$ $\tau = \frac{h^2}{6}\frac{d^3\phi(1)}{dx^3}$

b) $\underbrace{-\frac{u}{2h}}_{L}\phi_{i-1} \underbrace{-\lambda}_{D}\phi_i + \underbrace{\frac{u}{2h}}_{R}\phi_{i+1} = 6x_i$

$x=0$ $\quad \phi_0 = 0$

$x=1$ $\quad \frac{\phi_{N+1} - \phi_{N-1}}{2h} = 0 \Rightarrow \phi_{N+1} = \phi_{N-1}$

$\Rightarrow -\lambda\phi_N = 6x_N = 6$

$$\left[\begin{array}{ccccc|c} 1 & & & & & 0 \\ L & D & R & & & 6x_1 \\ & \ddots & & & & \\ & & L & D & R & 6x_{N-1} \\ & & & & -\lambda & 6 \end{array}\right]$$

c) diag dominant : $|D| \geq |L| + |R| \Rightarrow |\lambda| > \left|\frac{u}{2h}\right| + \left|\frac{u}{2h}\right| = \frac{|u|}{h}$

this implies unique solution and wiggle free solution

grid refinement $\Rightarrow h\downarrow \Rightarrow$ makes situation worse

2 slides p85

1 a) $R = \frac{D\,\delta t}{h^2} = \frac{1/40\ \delta t}{(1/10)^2} < \frac{1}{2} \longrightarrow \delta t < \frac{1}{5}$

$\phi(x,t) = x$ is exact sol. $\Longrightarrow$ no errors during time marching
no error accumulation $\longrightarrow$ no δt limit

b) $R = \frac{D\,\delta t}{h^2} = \frac{\frac{1}{40}\ \frac{1}{10}}{(1/10)^2} = 1/4$

$$\phi_i^{n+1} = R\,\phi_{i-1}^{n} + (1-2R)\,\phi_i^{n} + R\,\phi_{i+1}^{n} + 100\,x_i\,\phi_i^{n}\cdot\delta t$$

at $x = \frac{1}{5} \Rightarrow = \frac{1}{4}\left(\frac{1}{10}\right)^2 + \frac{1}{2}\cdot\left(\frac{2}{10}\right)^2 + \frac{1}{4}\left(\frac{3}{10}\right)^2 + 100\cdot\frac{1}{5}\cdot\left(\frac{1}{5}\right)^2\cdot\frac{1}{10}$

$= \frac{1}{8}$

c) $\frac{\phi_i^{n+1} - \phi_i^{n}}{\delta t} + \alpha\,\frac{\phi_i^{n+1} - \phi_{i-1}^{n+1}}{h} = D\,\frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{h^2} + \beta x_i \phi_i^{n+1}$.

$\Longrightarrow -R\,\phi_{i-1}^{n+1} + (1+2R)\,\phi_i^{n+1} - R\,\phi_{i+1}^{n+1} = \phi_i^{n}$
$-\frac{\alpha\,\delta t}{h}\phi_{i-1}^{n+1} \quad + \frac{\alpha\,\delta t}{h}\,\phi_i^{n+1}$

$-\beta x_i \delta t\,\phi_i^{n+1}$

$\Longrightarrow$

$R = \frac{1}{4}$ (see (b)) $\qquad \frac{\alpha\,\delta t}{h} = \frac{1\ \ 1/10}{1/10} = 1 \qquad \beta x_i\,\delta t = 100\,\frac{1}{5}\,\frac{1}{10} = 2$

$\left(-\frac{1}{4} - 1\right)\phi_{i-1}^{n+1}$	$+\left(1 + \frac{1}{2} + 1 - 2\right)\phi_i^{n+1}$	$-\frac{1}{4}\,\phi_{i+1}^{n+1}$	$\left(\frac{1}{5}\right)^2$
$-\frac{5}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{25}$

2 <u>slides</u> p68

1a) $(v, f - Au) = 0 \Longrightarrow (v, -u'' + 2u) = (v, 3x)$

$$\Longrightarrow -\int_0^2 v u'' dx + \int_0^2 v.(2u) dx = \int_0^2 v.(3x) dx$$

$$\Longrightarrow \underbrace{-vu'\Big|_0^2}_{=0} + \underbrace{\int_0^2 v'u' dx + \int_0^2 v.(2u) dx}_{=a(v,u)} = \underbrace{\int_0^2 v.(3x) dx}_{=F(v)}$$

Find $u \in \hat{V} = \{H'[0,2] \mid v(0) = 0\}$ s.t. $\forall v \in \hat{V}: a(v,u) = F(v)$

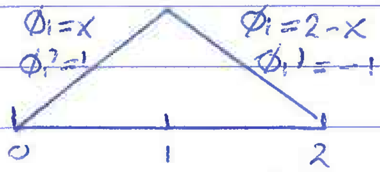
b) $a(u,u) = \int_0^2 (u')^2 dx + \int_0^2 2u^2 dx \geqq 0$

$a(u,u) = 0$ if $u' = 0$ and $u = 0$ which holds for $u \equiv 0$ $\Big\}$ $a(,)$ pos. def.

$|F(v)| = |(v, 3x)| \leq \|v\| \|3x\| \leq \sqrt{\|v\|^2 + \|v'\|^2} \|3x\|$

$3x$ max 6 on $[0,2] \longrightarrow \leq \|v\|_{H_1} . 6$ $\Big\}$ $F(v)$ bounded

c) $A_{1,1} = \int_0^2 a(\phi_1, \phi_1) dx$



$$= \int_0^1 \phi_1' \phi_1' dx + \int_0^1 \phi_1 2 \phi_1 dx + \int_1^2 \phi_1' \phi_1' dx + \int_1^2 \phi_1 2 \phi_1 dx$$

$$= \int_0^1 1^2 dx + \int_0^1 2x^2 dx + \int_1^2 (-1)^2 dx + \int_1^2 2(2-x)^2 dx$$

$$= x\Big|_0^1 + \frac{2}{3}x^3\Big|_0^1 + x\Big|_1^2 + -\frac{2}{3}(2-x)^3\Big|_1^2 = 1 + \frac{2}{3} + 1 + \frac{2}{3} = 3\frac{1}{3}$$

$$b_1 = \int_0^2 F(\phi_1) dx = \int_0^1 \phi_1 . 3x\, dx + \int_1^2 \phi_1 . 3x\, dx$$

$$= \int_0^1 x(3x) dx + \int_1^2 (2-x) . 3x\, dx$$

$$= x^3\Big|_0^1 + (3x^2 - x^3)\Big|_1^2 = (1-0) + (4-2) = 3$$